Prediction of Long Term Evolution of Satellite Orbit

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Agenda

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Coordinate Frames

Force Model

Uncertainty Analysis

Numerical Integrators

Earth's Gravitational Potential

Surface Forces

- Atmospheric Drag
- Solar Radiation Pressure

Code Details

Results

Conclusion

Future Scope

Introduction

- Evolution of satellite orbit refers to:
  - Given position, attitude and their rate vectors at t_0
  - Obtain position at t_1
  - Obtain attitude at t_1
  - t_0 is generally less than t_1
- Depending on the difference between t_1 and t_0, and the accuracies demanded, various effects have to be considered.
- Principally, the orbit of a satellite is determined by the gravitational effect of earth on the satellite.
- Other effects act as perturbations and deviate the satellite from this orbit.

Problem Statement

- To develop a C++ code to simulate the orbit of the spacecraft for long duration missions with minimal deviation.
- To account for every force which can cause significant deviation over long duration.
- To implement a robust numerical integration scheme, to minimize the numerical error accumulation over long duration.

Earth Centered Inertial

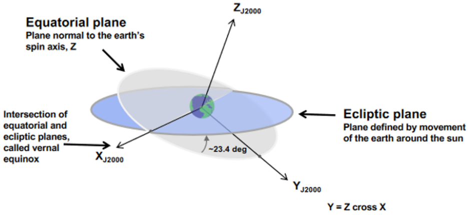

Figure: Earth Centered Inertial Frame

- Centered at the center of the earth.
- X-axis towards vernal equinox, Z-axis towards the north pole, Y-axis - completes the right handed system.
- The calculations are done in this frame.
- Ephemeris data in this frame.

Earth Centered - Earth Fixed

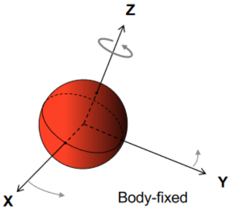

Figure: Earth Centered- Earth Fixed Frame

- Centered at the center of the earth
- Rotates with earth.
- X-axis towards Greenwich Meridian, Z-axis towards the north pole, Y-axis - completes the right handed system.
- To calculate geopotential data.
- To calculate ground track

S/C Centered ECI oriented

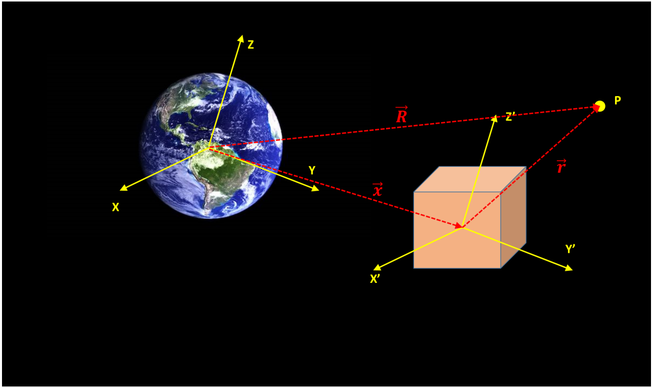

Figure: S/C Centered Inertial frame

- Centered on Center of Mass of the Spacecraft.
- Orientation same as that of ECI frame.

S/C Centered Inertial (Contd.)

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- Translated from ECI frame by the vector position of satellite in ECI frame
- Serves as the reference for calculating the orientation of satellite.
- For calculating position of bodies w.r.t. spacecraft.

S/C Centered Body Fixed

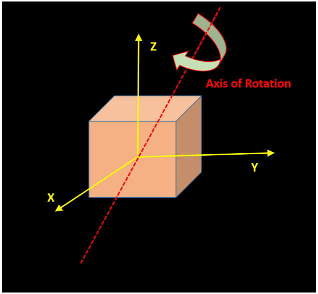

Figure: S/C Centered Body Fixed frame

- Centered at the center of mass of the spacecraft.
- Rotates with the body.
- Gives the orientation of the spacecraft.
- Useful to calculate SRP and Atmospheric Drag

Tranformations

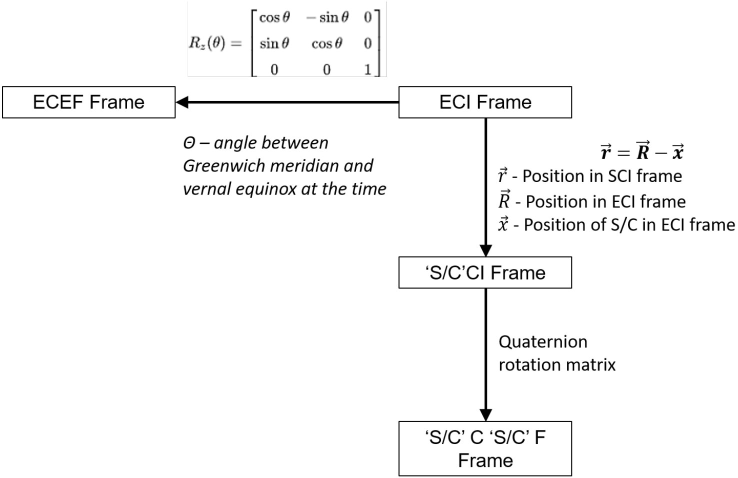

Figure: Transformation Scheme

CSPICE

- Toolkit by NASA's Navigation and Ancillary Information Facility.
- A C-language library.
- Used to access the ephemeris data.
- Also used in frame conversion from ECI to ECEF and vis-versa.

GMAT

- General Mission Analysis Tool developed by NASA.
- Free and Open Source.
- Used for validation of our program.
- Uses BOOST Library for numerical integration.
- Uses CSPICE for ephemeris data.
- Force models almost same as used in the code developed.

Major Forces

- Gravitational force:
  - Earth - Principle force acting on the satellite
  - Sun and Moon - Also called luni-solar perturbations, are important for medium and long-term evolution
  - Other planets - Only have to be considered if very accurate models are desired. Jupiter and Venus are the major perturbers
- Atmospheric drag - Most important for low earth satellites.
- Solar radiation pressure

Minor forces

- Earth radiation pressure - Due to radiation emitted by earth
- Is of two types:
  - Shortwave optical - Due to earth's albedo
  - Longwave infrared - Due to blackbody radiation of earth
- Earth tides - Secondary effect due to gravitation of moon and sun
- Falls off as $\frac{1}{r^4}$
- Relativistic effects - Proportional to v^2/c^2
- Acceleration due to this is approximately $5x10^{-9} m/s^2$

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Force model comparison

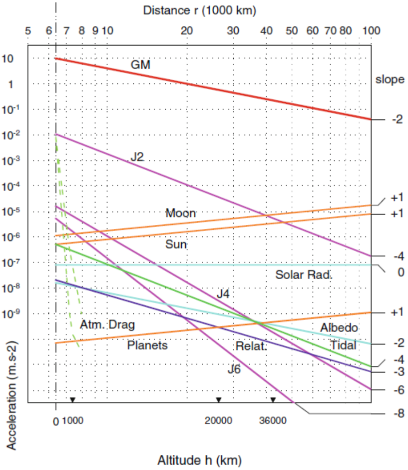


Figure: Order of magnitude of various perturbations of a satellite orbit

Uncertainty Analysis

- Positional Uncertainty
- Due to impossibility of knowing the exact position of the satellite
- Velocity Uncertainty
- Due to uncertainty in knowing exact velocity of satellite

δP_{in} **(m)**	δV_{in} **(m/s)**	δP_{final}
10	0	12.09 km
1	0	1.20 km
0.1	0	0.12 km
0	1	2007.3 km
0	0.1	200.7 km
0	0.01	20.7

Numerical Integrators - Single Step/Multi Step

Single step methods

- Compact and easy to use compared to multi-step methods.
- No requirement of previous history of data.
- A new stepsize may be used in each step.
- Higher number of function evaluations.
- Higher order methods are computationally expensive.

Multi step methods

- More efficient compared to single-step methods.
- Requirement of previous history of data.
- Reduced number of function evaluation per time step.
- Its not self-starting in nature.
- Needs to be re-initialized in case of discontinuities.

Explicit methods

- Simple to set up and program.
- Small time steps required for stability.
- Many computing steps required for a given time interval t.
- Fast computation per integration steps.
- Due to the limit on step size, can't extensively be applied to stiff systems.

Implicit methods

- Complicate to set up and program.
- More robust as stability is maintained for larger time steps.
- Requires less computing steps for a given interval t.
- Large computing time per integration step.
- Generally applied to solve stiff system.

Numerical Integrator - Implemented

- Runge-Kutta-Fehlberg 4(5), was implemented with adaptive step size.
- Advantages
  - Simple to code.
  - Explicit method.
  - Reinitialization not required.
- Disadvantages
  - Can be unstable.
  - More function evaluation required.

Earth’s Gravitational Potential

- For the most part, earth’s gravitation is a **radially symmetric force** following the **inverse square law**.
- For accurate **medium** and **long** term projections deviations from this have to be considered.
- Since gravity is a conservative force, it is possible to define a potential for this force.
- This potential can be expanded in the form of **spherical harmonics**.
- The first term gives the radially symmetric force whereas higher order terms give the deviations

Gravitational Model of Earth

- Gravitational potential of earth is expressed in terms of Legendre polynomials
- The form of the equation is:

$$U = \mu_e \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{R_e}{r^n} P_{nm}(sin\phi)(C_{nm}cos(m\lambda) + S_{nm}sin(m\lambda)) \quad (1)$$

where P_{nm} are the Legendre polynomials.

- Due to the large variation in C_{nm}, they are generally stored as normalized coefficients given by:

$$\bar{C}_{nm} = \sqrt{\frac{(n+m)!}{(2-\delta_{0m})(2n+1)(n-m)!}} C_{nm} \quad (2)$$

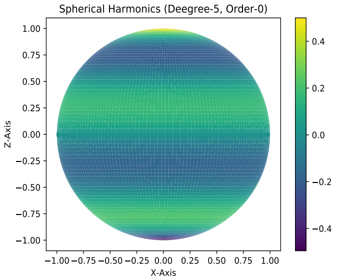
Gravitational Model of Earth (Contd.)

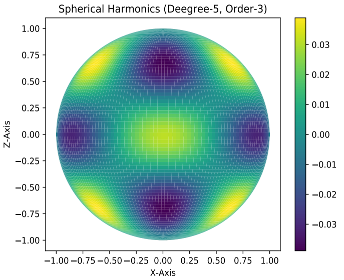
- $\bar{C}_{nm}$ and $\bar{S}_{nm}$ were obtained from EGM2008.
- Gradient of the potential (in Cartesian coordinates) obtained via the method outlined by Cunningham(1970).
- First order partial derivatives are expressed as a series of recursive functions given by

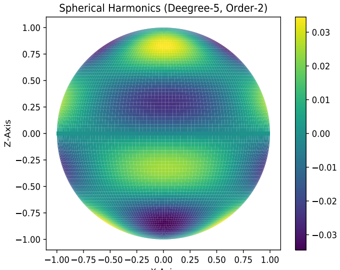
$$V_{nm} = \left(\frac{R_{e.}}{r}\right)^{n+1} P_{nm}(sin(\phi))cos(m\lambda) \quad (3)$$

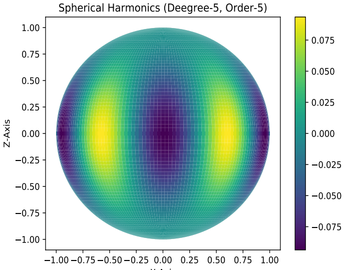
- For stability recursions, are calculated keeping m (or n) constant.
- Accelerations are obtained in ECEF-coordinate system.

Spherical Harmonics









Spherical Harmonics (contd.)

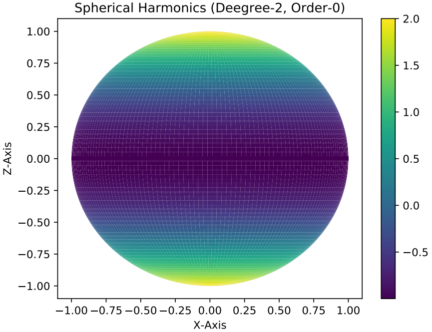


Figure: Main gravitation perturbation component J2

Luni-Solar Perturbations

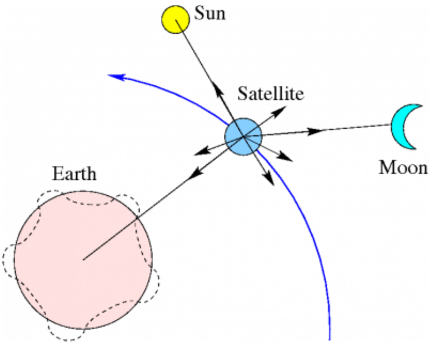


Figure: Luni-Solar effects on Spacecraft

- These refer to the perturbation in satellite orbit due to the combined effect of sun and moon
- Both can be considered point masses for calculation of these perturbations

Luni-Solar Perturbations (Contd.)

- Precision of position of these bodies is required
- Can be obtained by ephemeris data published by JPL
- These are in the form of Chebyshev polynomials
- For very accurate predictions, other planet's perturbations have to be considered

Body	**Acceleration (m/s^2)**	δP **(8-days)**
Sun	$5.11 * 10^{-7}$	122 km
Moon	$1.07 * 10^{-6}$	256 km
Venus	$5.75 * 10^{-11}$	13 m
Jupiter	$6.49 * 10^{-12}$	1.55 m

Atmospheric drag

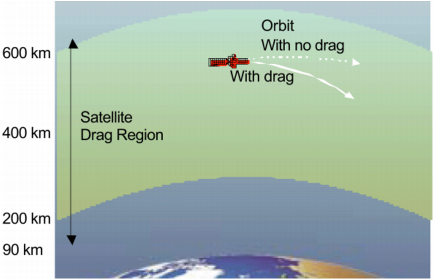


Figure: Effect of atmospheric drag on spacecraft

- Is very important for low altitude satellites (<600km)
- Accurate modeling is difficult because:
  - The atmosphere shows both spatial and temporal variation
  - Interaction of neutral and charged particles with spacecraft.
  - Varying attitude of non-spherical spacecrafts
- The flow can be modeled as free molecular because of high Knudsen Number (>10).

Atmospheric drag
Density Models

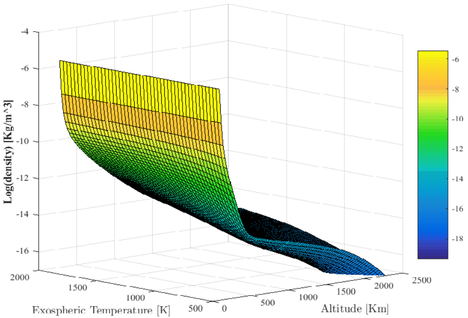


Figure: Density variation according to Jacchia-71 density model

- Various models of atmospheric drag have been established:
  - Jacchia Model family (**Jacchia 71**, Jacchia-77, JB 2008)
  - MSIS Model (MSISE-86, MSISE-90, NRLMSISE-00)
  - Harris-Priester model

Jacchia-Gill (1996) was employed for density correction.

- It uses a bi-polynomial approximation of the Jacchia 1971 standard density model.
- Computational time reduced by factor of 9 compared to J71.
- Typical differences with J71 are 2% and the maximum deviation is 8%.

Density Corrections

- Density correction due to geomagnetic effect of earth.
- Semi-annual density variation in thermosphere due to spatial and temporal variations.
- Seasonal-latitudinal correction for density variations.

Atmospheric drag

Satellite acceleration due to drag is written as :-

$$\ddot{r}_{drag} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2 e_v \quad (4)$$

where
$2.0 < C_D < 2.3$, for non-spherical convex spacecrafts,
$v_r = v - \omega r$, where ω = Earth's angular velocity
A = projected spacecraft's area

Satellite torque due to drag is written as :-

$$T = \vec{L} \times \vec{F}_{drag} \quad (5)$$

Here

$$\vec{L} = \vec{X}_{cp} - \vec{X}_{cg}$$

CP = Centre of Pressure
CG = Centre of mass

Solar Radiation Pressure

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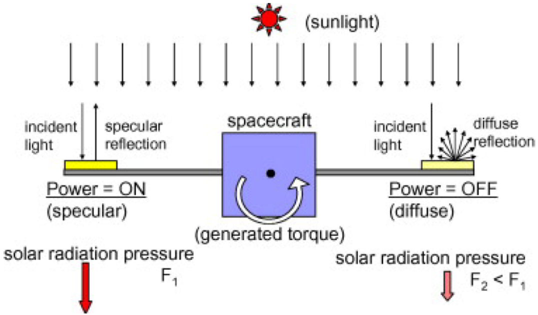


Figure: Effect of Solar radiation pressure on spacecraft

- Surface force dependent upon the ratio of surface area to mass of spacecraft
- A function of type of surface (absorbing/reflecting)
- Can change the attitude of the spacecraft
- Eclipse of satellite due to earth and moon to be accounted

Solar Radiation Pressure (contd.)

Satellite acceleration due to drag is written as :-

$$\ddot{r}_{srp} = -\nu P_{sun} \frac{1AU^2}{r_{sun}^2} \frac{A}{m} cos(\theta)[(1-\epsilon)e_{sun} + 2\epsilon cos(\theta)n] \quad (6)$$

where ν is shadow function
$P_{sun} \equiv 4.56e^{-6}\ Nm^{-2}$ (SRP at 1AU)
θ = Angle between unit area normal and Sun's unit vector

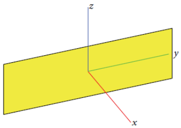
Computation of $\vec{F}_{srp}$ requires modelling of :-

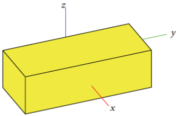
- Satellite's orbit as it affect shadow function as well as P_{sun}.
- Satellite's attitude as the incident angle (θ) of satellite varies with it.
- Satellite's geometry.

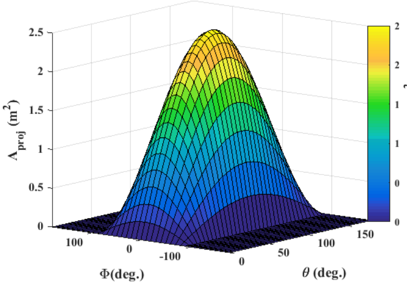
Solar Radiation Pressure
Geometry Models

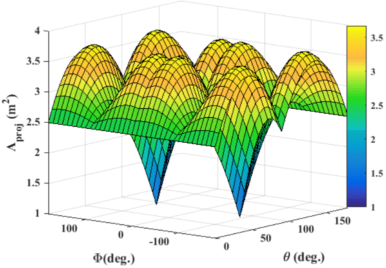
Satellite's geometry can be modelled as :-

- Cannonball model
- Disk model
- Box model
- Wing Box model









Dual Cone shadow model was used for the computation of eclipse conditions.

- Spherical Earth Conical shadow model was used.
- $\nu = 0$, when satellite in umbra
- $0 < \nu < 1$, when satellite in penumbra
- Occultations of Sun by Earth as well as Moon modelled.

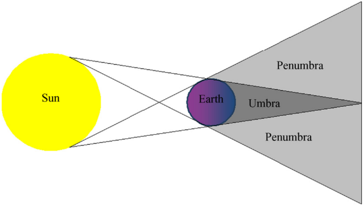


Figure: Umbra and penumbra

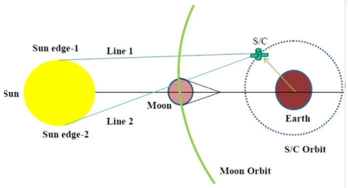


Figure: Satellite's Eclipse model

Code Details

- The code has been written with C++.
- An input file gives the initial conditions and other constants.
- The force models have been implemented via a class called *force_mod*.
- Numerical integrator implementation
  - In-house Runge-Kutta-Fehlberg 4(5)
  - BOOST library

Code Flowchart

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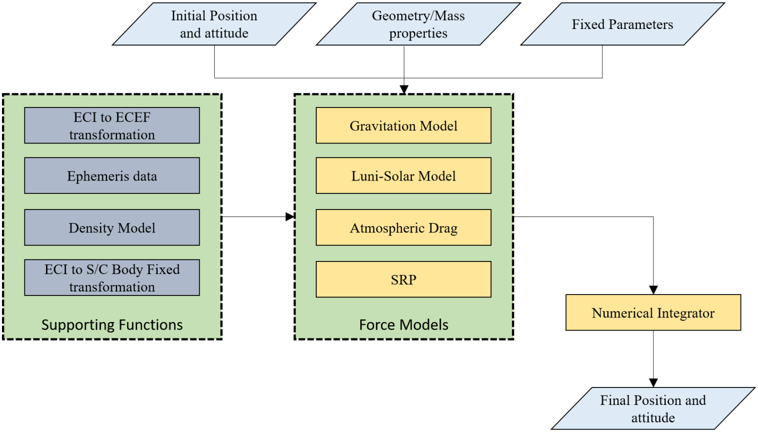

Figure: Flowchart depicting the working of the code

Satellite Initial Conditions and Ballistic Properties

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- Initial positions (6660136.3 , 0 , 0) m.
- Initial Velocity (0,-889.18,7684.92) m/s.
- Initial time (2019/09/23 05:03:40) UTC Gregorian.
- Ballistic Properties
  - Mass - 850 Kg
  - C_D - 2.2
  - Area - 1 m^2
  - Emissivity(ϵ) - 0.8

Results

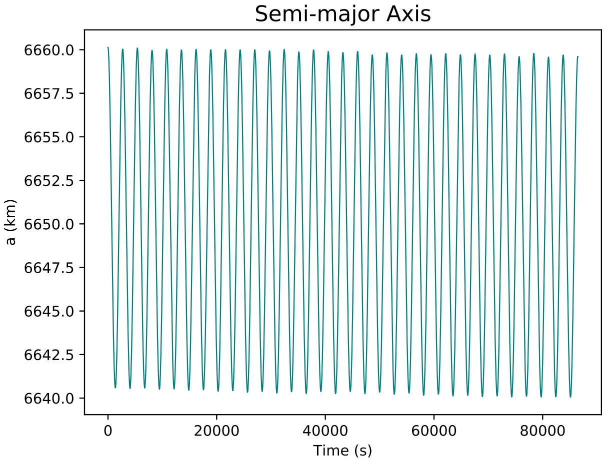

Figure: Variation of semi-major axis with time

Results

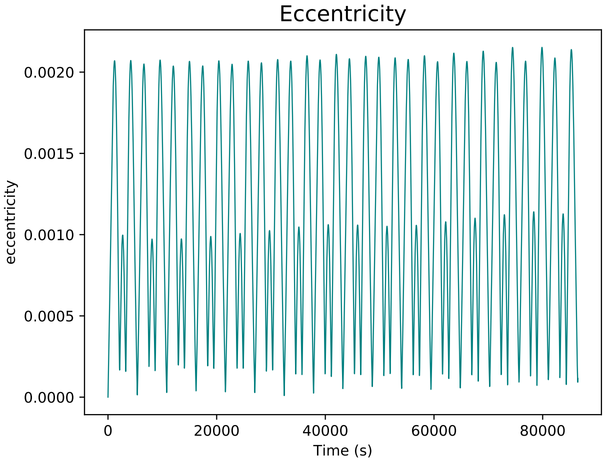

Figure: Variation of eccentricity with time

Results

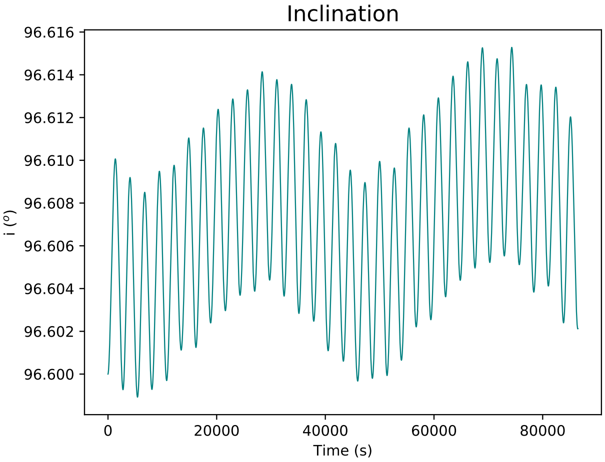

Figure: Variation of inclination with time

Results

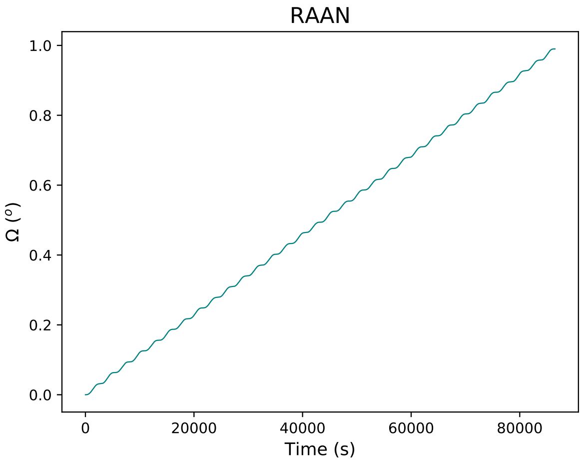

Figure: Variation of right ascension of ascending node with time

Results

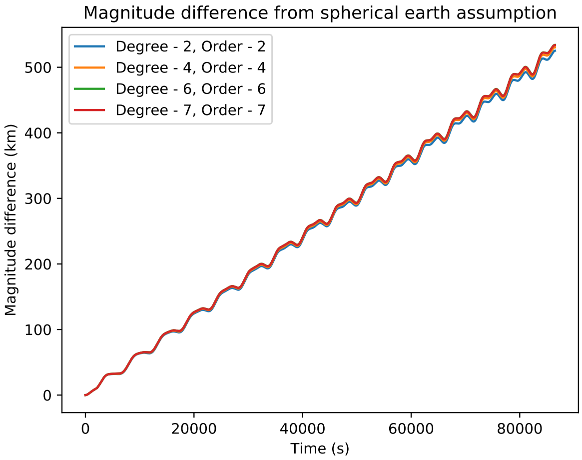

Figure: Effect of including higher order and degree of geopotential

Results

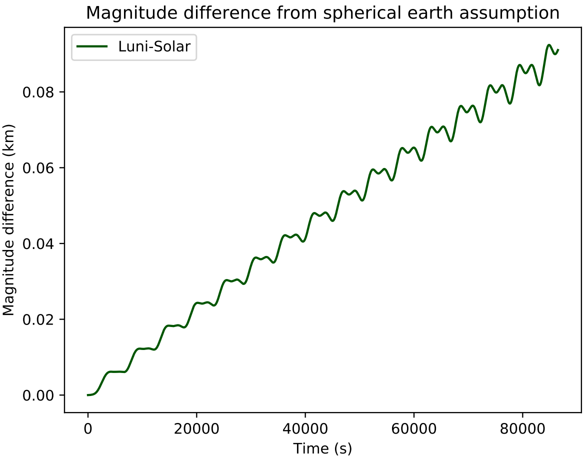

Figure: Magnitude of difference in position with and without Luni-Solar Perturbation

Results

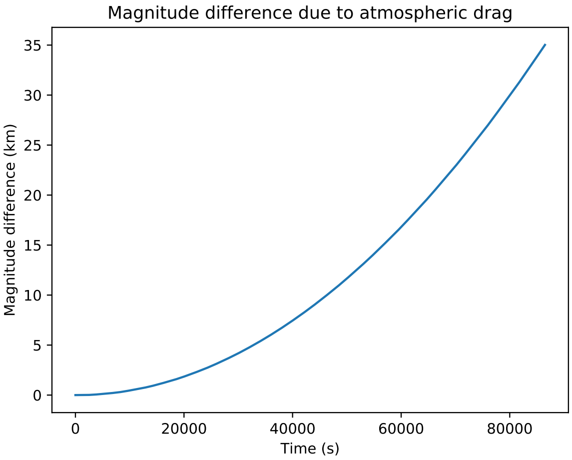

Figure: Magnitude of difference in position with and without Atmospheric Drag

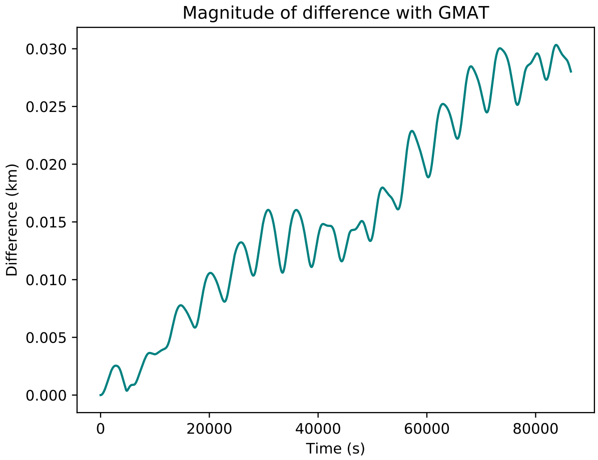

Figure: Magnitude of difference in position calculated from the code and GMAT, without atmospheric drag

Results

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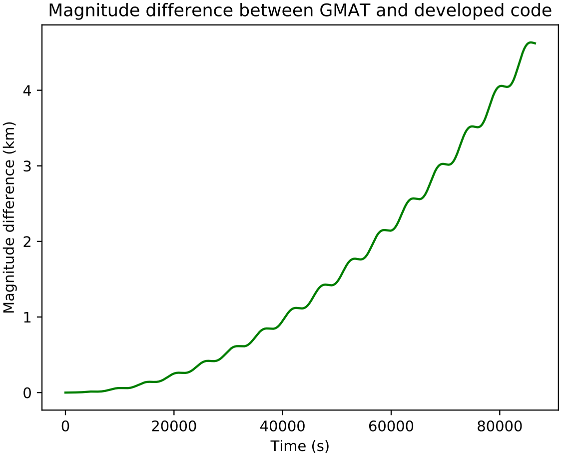

Figure: Magnitude of difference in position calculated from the code and GMAT

Conclusions

- A C++ code was written for long term orbit simulation, with in-house numerical integrators.
- Post-processing of the data was done via Python.
- Reasonable agreement with GMAT's data was found.
- With all major forces considered, the magnitude difference in position was found to be 4.5 km.
- This is attributed to difference in density models implemented. (Jacchia-Roberts/Jacchia-Gill).
- Exospheric density shows significant spatial and temporal variation which requires short term models.
- Without considering atmospheric drag, the difference was found to be approximately 30m.

Future Scope

- Inclusion of minor forces.
- Multiple classes of density and SRP models may be implemented.
- More complex geometries can be handled.
- Capability to model with multiple frames of reference.
- Integrate the post-processing module with the code.
- Development of GUI interface.

Videos

Thank you